UDC 005.8:658

A. V. SIMAK, V. I. DUBROVIN
National University Zaporizhzhia Polytechnic

LOGISTICS OF CAPACITATED TRANSPORTATION PROBLEMS

In the context of modern global logistics, enterprises face increasing challenges in managing transportation costs efficiently under strict infrastructure constraints. The classical transportation problem, while fundamental to operations research, assumes ideal conditions–unlimited route capacities and direct shipments–that rarely exist in practice. This discrepancy leads to theoretical models that yield «optimal» solutions which are unimplementable in real-world scenarios involving road repairs, limited warehouse throughput, or specific delivery priorities. This study addresses the critical need for adapting mathematical optimization models to these realistic constraints. The purpose of the research is to analyze the impact of additional constraints–specifically arc capacity limits and priority servicing of specific destinations–on the optimal distribution plan and total transportation costs. The study aims to quantify the "cost of constraints" and demonstrate how mathematical modeling can support decision-making in disrupted supply chains. The research methodology is based on mathematical modeling of transportation networks using linear programming. The study utilizes the LINDO (Linear, Interactive, and Discrete Optimizer) software environment to solve optimization tasks. The modeling process involves two scenarios: a basic unconstrained transportation problem and a capacitated variation with priority subsets. The approach integrates theoretical foundations established by L. Kantorovich and F.L. Hitchcock with modern algorithmic solutions for capacitated networks. Additionally, the study employs dual problem analysis to determine shadow prices, providing an economic interpretation of the limiting factors. The results of the numerical experiment, conducted on a balanced transportation network with four sources and five destinations, demonstrate that introducing capacity constraints on specific arcs and imposing priority delivery conditions significantly alters the optimal distribution plan. The system is forced to reroute flows through more expensive paths to satisfy the new boundary conditions. In the case study, these constraints led to an increase in total transportation costs from 195 to 240 monetary units, representing a 23 % rise in expenses. This quantitative finding validates the theoretical assumption that restrictions on the feasible region in linear programming generally worsen the objective function value. Sensitivity analysis further revealed that specific «bottleneck» routes have high shadow prices, indicating where infrastructure investment would yield the highest return. The practical value of this research lies in providing a proven methodology for simulating logistical bottlenecks. By calculating the cost difference between unconstrained and constrained models, logistics managers can make informed financial decisions–for example, determining whether investing in infrastructure to remove a bottleneck is more cost-effective than paying higher transportation costs. The proposed model is adaptable for multi-product and multi-stage logistics networks and serves as a basis for decision support systems in regional logistics centers. The findings of this study also provide a foundation for future research into dynamic routing algorithms under uncertainty conditions and autonomous transport integration.

Keywords: *logistics optimization, capacitated transportation problem, linear programming, LINDO software, operations research, supply chain constraints, transportation modeling.*

А. В. СІМАК, В. І. ДУБРОВІН
Національний університет «Запорізька політехніка»

ЛОГІСТИКА ТРАНСПОРТНИХ ЗАДАЧ З ОБМЕЖЕНОЮ ПРОПУСКНОЮ ЗДАТНІСТЮ

В умовах сучасної глобальної логістики підприємства стикаються зі зростаючими викликами щодо ефективного управління транспортними витратами за жорстких інфраструктурних обмежень. Класична транспортна задача, будучи фундаментальною для дослідження операцій, передбачає ідеальні умови – необмежену пропускну здатність маршрутів та прямі поставки, – які рідко існують на практиці. Ця невідповідність призводить до того, що теоретичні моделі пропонують «оптимальні» рішення, які неможливо реалізувати в реальних сценаріях, пов'язаних з ремонтом доріг, обмеженою пропускною здатністю складів або специфічними пріоритетами доставки. Це дослідження присвячене актуальній проблемі адаптації математичних моделей оптимізації до цих реалістичних обмежень. Метою дослідження є аналіз впливу додаткових обмежень – зокрема лімітів пропускної здатності дуг та пріоритетного обслуговування окремих пунктів призначення – на план оптимального розподілу та загальні транспортні витрати. Дослідження має на меті кількісно оцінити «вартість обмежень» та продемонструвати, як математичне моделювання може підтримувати прийняття рішень у порушених ланцюгах постачання. Методологія дослідження базується на математичному моделюванні транспортних мереж з використанням лінійного програмування. Для вирішення оптимізаційних задач використано програмне середовище LINDO. Процес моделювання включає два сценарії:

базову транспортну задачу без обмежень та її варіацію з обмеженою пропускною здатністю та пріоритетними підмножинами. Підхід інтегрує теоретичні основи, закладені Л. Канторовичем та Ф.Л. Гічкоком, із сучасними алгоритмічними рішеннями для мереж з обмеженнями. Додатково застосовано аналіз двоїстої задачі для визначення тіньових цін та економічної інтерпретації обмежень. Результати чисельного експерименту, проведеного на збалансованій транспортній мережі з чотирма джерелами та п'ятьма пунктами призначення, демонструють, що введення обмежень пропускної здатності на конкретних маршрутах та встановлення умов пріоритетної доставки суттєво змінює план оптимального розподілу. Система змушена перенаправляти потоки через дорожчі маршрути, щоб задовольнити нові граничні умови. У досліджуваному випадку ці обмеження призвели до зростання загальних транспортних витрат зі 195 до 240 грошових одиниць, що становить підвищення витрат на 23 %. Цей кількісний результат підтверджує теоретичне припущення, що звуження області допустимих рішень у лінійному програмуванні зазвичай погіршує значення цільової функції. Аналіз чутливості також виявив, що конкретні маршрути-»вузькі місця» мають високі тіньові ціни, вказуючи на пріоритетність інвестицій в інфраструктуру. Практична цінність дослідження полягає у наданні перевіреної методології для моделювання логістичних «вузьких місць». Розраховуючи різницю у витратах між моделями з обмеженнями та без них, менеджери з логістики можуть приймати обґрунтовані фінансові рішення – наприклад, визначати, чи є інвестиції в інфраструктуру для усунення вузького місця більш рентабельними, ніж оплата вищих транспортних витрат. Запропонована модель може бути адаптована для багатопродуктових логістичних мереж та слугувати основою для систем підтримки прийняття рішень у регіональних логістичних центрах. Результати цього дослідження також створюють основу для майбутніх досліджень алгоритмів динамічної маршрутизації в умовах невизначеності.

Ключові слова: *оптимізація логістики, транспортна задача з обмеженою пропускною здатністю, лінійне програмування, програмне забезпечення LINDO, дослідження операцій, обмеження ланцюга постачання, моделювання перевезень.*

Statement of the problem

In the modern global economy, logistics plays a pivotal role in the efficiency of enterprises. The ability to transport goods from producers to consumers with minimal time and financial costs is a key factor in competitiveness. At the heart of logistical planning lies the Transportation Problem (TP), a special class of linear programming problems. However, the classical TP assumes ideal conditions: unlimited capacity on routes, a single type of product, and direct shipments. In practice, logistics managers face numerous restrictions: roads have weight limits (capacitated arcs), goods may need to pass through intermediate hubs (transshipment), or specific customers may require priority delivery. Ignoring these constraints leads to «optimal» solutions that are unimplementable in reality. The relevance of this study is driven by the need to adapt classical mathematical models to the rigid constraints of the current logistical environment, particularly in conditions where infrastructure may be limited or damaged.

Analysis of remaining research and publications

The mathematical foundation of optimal resource allocation was significantly advanced by the Leonid Kantorovich, who was awarded the Nobel Prize in Economics for his contributions to the theory of optimum allocation of resources [1; 2]. Similarly, F.L. Hitchcock formalized the distribution of a product from several sources to numerous localities, creating the classical formulation of the TP [3]. Early works by Dantzig provided the simplex method and the modified distribution method (MODI) for solving basic TPs [4]. However, as supply chains grew more complex, researchers developed variations to address specific constraints. The Capacitated Transportation Problem (CTP), where flow on specific arcs is bounded by an upper limit, forces the solution away from the most direct routes. Research by Khurana highlights that capacity constraints often increase costs but ensure feasibility [5]. The Transshipment Problem expands the standard TP by treating both sources and destinations as intermediate nodes [6]. Multi-product variations [7], Production-Transportation models [8], and network allocation problems [9] further integrate complexity. Recent literature also explores heuristic approaches for solving unbalanced and multi-objective transportation problems in large-scale networks [10; 11].

In the context of Ukraine's post-war recovery, the relevance of Capacitated Transportation Problems increases significantly. The destruction of transport infrastructure (bridges, roads, railway

https://doi.org/10.32782/mathematical-modelling/2026-9-1-34 ISSN 2618-0332 (print), 2618-0340 (online)

nodes) creates severe capacity constraints ($d_{ij} \to 0$ or significantly reduced). Furthermore, the need to prioritize supplies to critical regions or military units perfectly matches the "Priority Subsets" constraints modeled in this study. Therefore, the mathematical apparatus of CTP becomes a key tool for national logistics planning in crisis conditions.

The aim of the study

The aim of this article is to analyze the theoretical variations of the transportation problem (including production, multi-stage, and capacitated types) and to practically demonstrate the impact of «bottleneck» constraints on the optimization result using the LINDO software.

Presentation of the main research material

The study utilizes mathematical modeling of linear programming problems [12]. To solve the modeled scenarios, the LINDO (Linear, Interactive, and Discrete Optimizer) software package is used, which allows for the input of objective functions and constraints in a straightforward algebraic format [13]. We consider a transportation problem with m sources and n destinations. To visualize the problem statement, a graphical representation of the transportation network is constructed (Fig. 1). The nodes on the left represent the supply sources, and the nodes on the right represent the demand destinations. The edges connecting them represent the possible transportation routes, each characterized by a specific cost c_{ij} and capacity c_{ij}.

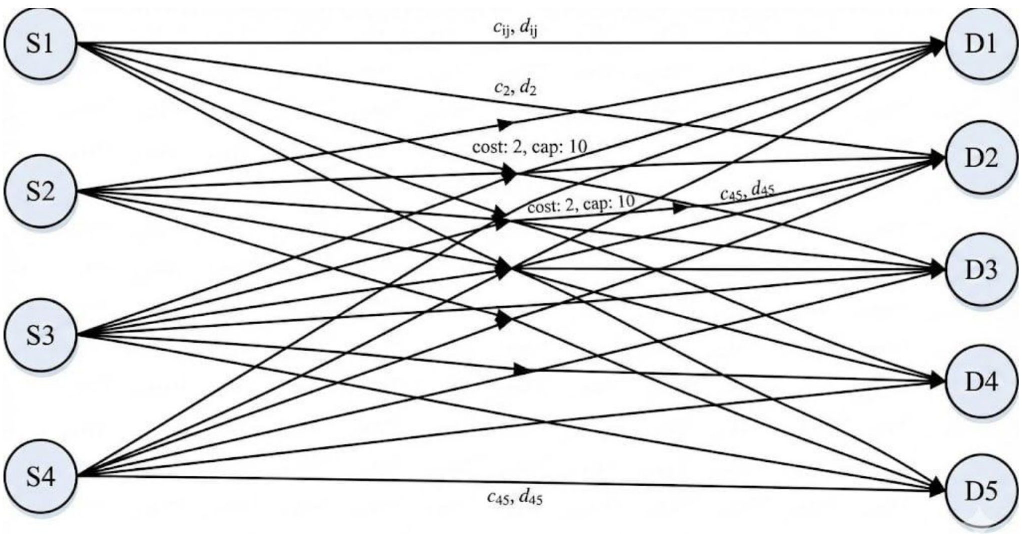

Fig. 1. Graph of the balanced transportation network model

Let a_i be the supply at source i, b_j be the demand at destination j, c_{ij} be the unit transportation cost, and x_{ij} be the quantity transported. The objective is to minimize total cost:

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min. \quad (1)$$

Subject to standard constraints: $\sum_{j=1}^{n} x_{ij} = a_i$, $\sum_{i=1}^{m} x_{ij} = b_{ij}$, $x_{ij} \geq 0$. In this study, we introduce two specific types of additional constraints:

1. **Arc Capacity Constraints:** The quantity transported on a specific route cannot exceed a limit d_{ij} ($x_{ij} \leq d_{ij}$).
2. **Priority Subsets:** A specific subset of sources S_k must satisfy the demand of a specific subset of destinations $D_k \left(\sum_{i \in S_k} \sum_{j \in D_k} x_{ij} = \theta_k \right)$.

The solution process in the LINDO environment utilizes the simplex method for linear programming, which allows for finding the global extremum of the objective function on a convex polyhedron of admissible solutions. For the Capacitated Transportation Problem (CTP), the standard simplex algorithm is modified to handle upper bound constraints efficiently without significantly increasing the dimensionality of the basis matrix. In matrix notation, the problem can be represented as follows: minimize $Z = CX$ subject to $AX = B$ and $0 \le X \le U$, where C is the row vector of costs, X is the column vector of variables, A is the matrix of coefficients for supply and demand constraints, B is the column vector of supply and demand values, and U is the matrix of upper bounds (capacities) for the arcs. The software implementation involves defining the objective function and constraints in a specific syntax that LINDO interprets. The priority subset constraints are modeled as additional linear equalities or inequalities added to the standard constraint set. This approach allows for flexible modification of the model structure without changing the underlying solution algorithm.

Dual Problem Formulation

Every linear programming problem has a corresponding dual problem. For the transportation problem, the dual variables u_i (associated with source constraints) and v_j (associated with destination constraints) play a crucial role in economic analysis. The dual problem can be formulated as:

$$W = \sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{m} b_j v_j \to \max. \quad (2)$$

Subject to: $u_i + v_j \le c_{ij}$ for all i, j.

In the context of our capacitated model, additional dual variables corresponding to the capacity constraints ($x_{ij} \le d_{ij}$) appear. These variables, often referred to as “shadow prices”, indicate the potential savings in total cost if the capacity of a specific “bottleneck” route were increased by one unit. This analysis is vital for identifying the most critical segments of the logistics network.

Results and Discussion

To demonstrate the impact of constraints, a numerical experiment was conducted. We analyze a balanced transportation problem with 4 sources (S1, S2, S3, S4) and 5 destinations (D1, D2, D3, D4, D5). The total supply and total demand are both 80 units.The unit transportation costs vary from 1 to 20 monetary units.

The initial parameters for the numerical experiment, including unit transportation costs c_{ij}, supply capacities a_i, and demand requirements b_j, are summarized in Table 1.

Scenario 1: Basic Transportation Problem (Unconstrained).Using LINDO to solve the basic model, the optimal distribution plan was generated. The solution utilized the most cost-effective routes (e.g., S1 to D2 with cost 1, S4 to D3 with cost 1).Total Minimum Cost: 195 units.

Table 1

Matrix of Transportation Costs and Capacities

Source \ Dest.	D1	D2	D3	D4	D5	Supply a_i
S1	20	1	3	4	5	20
S2	2	9	8	3	4	20
S3	3	6	5	2	8	20
S4	10	2	1	4	3	10
Demand b_j	35	37	37	13	15	Total: 80

Scenario 2: Capacitated and Priority Constraints.We introduced restrictions to simulate real-world disruptions:

https://doi.org/10.32782/mathematical-modelling/2026-9-1-34 ISSN 2618-0332 (print), 2618-0340 (online)

1. **Capacity Constraint:** Route S4 to D5 is limited to 5 units.
2. **Priority Constraint:** Demands for Destination 1 (D1) must be partially met by Source 4 (S4) with a limit of 2 units.
3. **Subset Constraint:** Sources S1 and S2 must prioritize satisfying the demand of D1 and D2 initially.The mathematical model was updated in LINDO.

The implementation of the constrained model in the LINDO syntax requires explicit definition of the new restrictions. Below is a fragment of the program code used to solve the Scenario 2 model:

```
MIN 20X11 + 1X12 + 3X13 + 4X14 + 5X15 + 2X21 + 9X22 +... + 3X45
SUBJECT TO
! Supply Constraints
2) X11 + X12 + X13 + X14 + X15 = 20
3) X21 + X22 + X23 + X24 + X25 = 20
...
! Demand Constraints
6) X11 + X21 + X31 + X41 = 35
...
! Additional Capacity Constraints
11) X45 <= 5
! Priority Subset Constraint (Source 4 for Dest 1)
12) X41 <= 2
! Priority Group Constraint (Sources 1,2 for Dest 1,2)
13) X11 + X12 + X21 + X22 >= 15
END
```

The solver recalculated the optimal flow to respect these boundaries. This forced the logistics plan to utilize more expensive routes because the cheapest routes were either “blocked” (capacity reached) or reserved for other priorities.Result of Constrained Model: The optimal distribution changed significantly. The system had to reroute goods through expensive paths (e.g., using routes with costs of 8 or 10 units).New Total Cost: **240 units** (Table 2).

The analysis of the **shadow prices (dual values)** generated by LINDO provides further insight. In Scenario 2, the shadow price for the capacity constraint ($x_{45} \leq 5$) is non-zero, indicating that every additional unit of capacity on this route would significantly reduce the total cost. This mathematically confirms that this specific route is the system’s bottleneck.Furthermore, the priority constraint forcing specific sources to serve specific destinations acts as a penalty mechanism. By restricting the freedom of choice for the optimizer, the system loses the ability to leverage the global minimum of transportation costs, settling instead for a local optimum within the restricted feasible region.

Sensitivity Analysis and Reduced CostsAn important part of the optimization result is the analysis of reduced costs. For the routes that were not selected in the optimal solution (i.e., $x_{ij} = 0$), the reduced cost indicates how much the transportation cost c_{ij} on that specific route must decrease before it becomes economically viable to use it.Additionally, we analyzed the “Shadow Prices” for the imposed constraints. The LINDO report provided the following dual values for the critical constraints in Scenario 2 (Table 3).

The high shadow price for the S4 $\rightarrow$ D5 capacity constraint confirms that this arc is the primary bottleneck of the system. Investment in expanding this specific route would yield the highest return on investment (ROI) in terms of logistics cost reduction.

The increase in cost from 195 to 240 represents a **23 % increase** in logistical expenses purely due to constraints. This quantitative result validates the theoretical assumption that restrictions on the domain of feasible solutions in linear programming generally worsen (or at best equal) the value of the objective function. In the context of the Production-Transportation problem, this cost increase

https://doi.org/10.32782/mathematical-modelling/2026-9-1-34 ISSN 2618-0332 (print), 2618-0340 (online)

Table 2

Comparison of Optimization Results

Indicator	Scenario 1 (Unconstrained)	Scenario 2 (Constrained)	Deviation
Total Cost	195 units	240 units	**+23.1 %**
Used Routes	8 active routes	9 active routes	**+1 route**
Key Change	Utilized cheapest routes (c_{ij} = 1..3)	Forced to use expensive routes (c_{ij} = 8..10)	**Reallocation**
Bottleneck	None	Route S4 → D5 (saturated)	**Capacity limit hit**

Table 3

Shadow Prices for Critical Constraints

Constraint Type	Specific Constraint	Dual Value (Shadow Price)	Interpretation
Capacity Limit	Route S4 → D5	**–8.0**	Increasing capacity by 1 unit saves 8 monetary units
Priority Supply	Source S4 → D1 ≤ 2	**12.0**	Relaxing this forced supply saves 12 units per unit
Supply Balance	Source S3 Capacity	**0.0**	Source is not fully utilized or has alternatives

signals to the manufacturer that investing in infrastructure (to remove the bottleneck) might be more profitable than paying high transport costs [14; 15].

Conclusions

The study confirmed that the standard Transportation Problem is an idealization, while real-world logistics requires the use of the Capacitated Transportation Problem (CTP) model. Mathematical modeling using LINDO proved effective for solving both basic and constrained problems. The introduction of constraints on arc capacity and priority servicing led to a significant revision of the optimal plan. The cost analysis showed that imposing constraints increased the total transportation cost by approximately 23 % (from 195 to 240). This «cost of constraint» is a critical metric for logistics managers. Future research should focus on solving large-scale multi-product problems using heuristic algorithms, as classical linear programming may become computationally expensive for national-scale networks and autonomous transport integration.

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Simak Anzhelika Vitaliivna – Student at the Department of Software of the Zaporizhzhia Polytechnic National University. E-mail: kapiba@ukr.net, ORCID: 0009-0003-5153-5961.

Dubrovin Valerii Ivanovych – Candidate of Technical Sciences, Professor at the Department of Software of the Zaporizhzhia Polytechnic National University. E-mail: vdubrovin@gmail.com, ORCID: 0000-0002-0848-8202.

Сімак Анжеліка Віталіївна – студентка кафедри програмних засобів Національного університету «Запорізька політехніка». E-mail: kapiba@ukr.net, ORCID: 0009-0003-5153-5961.

Дубровін Валерій Іванович – к.т.н., професор кафедри програмних засобів Національного університету «Запорізька політехніка». E-mail: vdubrovin@gmail.com, ORCID: 0000-0002-0848-8202.

Дата першого надходження статті до видання: 09.04.2026
Дата прийняття статті до друку після рецензування: 11.05.2026
Дата публікації (оприлюднення) статті: 01.07.2026