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ANALYSIS OF DYNAMIC CHARACTERISTICS OF AN UNREGULATED OBJECT

The article studies an unregulated object and analyzes the dynamic structure of the object based on a steady-state signal. The first stage of the analysis is associated with general questions: based on a priori data on the object under study, one must first select one of the operator types, choosing functional, differential (ordinary, with a lagging argument with partial derivatives), integral or integro-differential operators. Then we limit the selected operator type. Taking into account more detailed a priori information, we limit ourselves to considering linear or weakly nonlinear operators with constant coefficients. Under such conditions, it is necessary to take into account not only the a priori properties of the analyzed object, but also the preliminary information obtained from the signal. Regularities in the behavior of the signal make it possible to ignore any class of operators as clearly not corresponding to the observed manifestations of the object. The development of methods for finding in a certain class an equation that has a given function as its solution relates to inverse problems of analysis. The direct scheme – to find the movement of an object of known structure under given conditions – has a narrower technical area of direct applications.

In the work, a general and fairly simple principle for describing a signal was formulated and, to some extent, substantiated. According to this basic position, the quantitatively significant and regularly manifested properties of a signal under given observation conditions are linked to each other by a certain dynamic structure of the object. The role of the object's movements, which are less significant under these conditions, as well as the role of the external environment, is reflected in this description by the force $F(t)$, which fluctuates in time and disturbs the dynamic system. The task of analyzing the dynamic structure of an object is reduced to assessing the numerical values of the coefficients $A_0^{(k)}, A_m^{(k)}$. A priori ideas about the dynamic structure of the analyzed object allow us to represent these coefficients in the form of unambiguous expansions, which are described in detail in the article. Based on the numerical processing of the signal $U(t; \mathbf{x})$, the dynamic characteristics of the unregulated object were analyzed.

Key words: dynamic system, trajectory, fluctuation disturbance, shift, correlation, unregulated object.

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АНАЛІЗ ДИНАМІЧНИХ ХАРАКТЕРИСТИК НЕРЕГУЛЬОВАНОГО ОБ'ЄКТА

У статті досліджено нерегульований об'єкт, проаналізовано динамічну структуру об'єкта за сигналом, що встановився. З першим етапом аналізу пов'язані загальні питання: за апіорними даними про досліджуваний об'єкт треба спочатку вибрати один із типів оператора, зупинившись на функціональних, диференціальних (звичайних, з аргументом з приватними похідними, що запізнюється), інтегральних або інтегро-диференціальних операторах. Потім обмежуємо вибраний тип операторів. Ураховуючи детальніші апіорні відомості, обмежуємося розглядом лінійних або слабонелінійних операторів з постійними коефіцієнтами. За таких умов необхідно враховувати не тільки апіорні властивості аналізованого об'єкта, а й попередню інформацію, одержувану із сигналу. Закономірності в поведінці сигналу дають змогу не враховувати якийсь клас операторів як явно не відповідний виявам об'єкта, що спостерігається. Розробка методів відшукування в певному класі рівняння, що має задану функцію своїм розв'язанням, належить до обернених задач аналізу. Пряма схема – знайти за заданих умов рух об'єкта відомої структури – має вузьку технічну сферу безпосередніх додатків.

У роботі сформульовано й певною мірою обґрунтовано загальний і досить простий принцип опису сигналу. Згідно із цим основним положенням спостереження, які кількісно суттєві й що регулярно проявляються за цих умов, властивості сигналу зв'язуються між собою деякою динамічною структурою об'єкта. Роль менш істотних за цих умов рухів об'єкта, як і роль зовнішнього середовища, відображає в цьому описі сила $F(t)$, що обурює динамічну систему, що флукує в часі. Завдання аналізу динамічної структури об'єкта зводиться до оцінки числових значень коефіцієнтів $A_0^{(k)}, A_m^{(k)}$. Апіорні уявлення про динамічну структуру об'єкта, що аналізується, дають змогу представити ці коефіцієнти у вигляді однозначних розкладів, які докладно описані в статті. З огляду на чисельну обробку сигналу $U(t; \mathbf{x})$, проаналізовано динамічні характеристики нерегульованого об'єкта.

Ключові слова: динамічна система, траєкторія, флуктуаційне збурення, зсув, кореляція, нерегульований об'єкт.

Problem Statement

At the research stage, it is advisable to consider a fairly complex object as unregulated, the connections of which with the external environment can greatly complicate the analysis. When studying an unregulated object, it is of great importance that signals recorded in sufficient detail always not only describe the behavior of the object as a whole, but also bear the “imprints” of individual movements of a large number of its similar microparts [1; 2]. The simplest mathematical model of the formation of the signal $U(t)$ in time t that takes this into account can be given the form of the equation $D_0[U] = F(t)$, where $F(t)$ are small short-correlated Gaussian fluctuations that disturb the steady-state dynamic change in the signal, which occurs according to the equation $D_0[U] = 0$ [1; 3]. The class of operators generating dynamic equations that dynamically approximate the observed manifestations of the analyzed object is determined based on a priori information about the latter, from analogous considerations that take into account the properties of objects, and also from the principle of simplicity of description, expressed in accordance with ideas about the essence of the problem.

Analysis of recent studies and publications

The analysis of dynamic characteristics of established signals is a fundamental task with wide application. Research in this area has been conducted for many decades, and many scientists have made significant contributions to it, such as N. Wiener, A. Kolmogorov, K. Shannon, L. Hume, R. Botha and others. Norbert Wiener's work became the basis for modern methods of analyzing random signals, which are of great importance for the study of established signals. Mathematician Kolmogorov A. is the author of the theory of stationary random processes; his works served as the basis for the mathematical apparatus used to analyze established signals. Claude Shannon's work is fundamental to the analysis of established signals, especially in the context of data transmission. Leonard Hume's ideas about inductive inference have found application in the analysis of established signals, where conclusions are drawn from observations about the properties of the signal-generating system. The work of mathematician Ralph Botha is devoted to the analysis of dynamic systems, including systems with established signals. Research into the dynamic characteristics of an unregulated object opens up new possibilities for developing effective methods for signal processing and solving a wide range of scientific and technical problems.

Purpose of the research

The aim of the study is to analyze the dynamic characteristics of an uncontrolled object near its ω -limit trajectory.

Presentation of the main research material

For different values of the parameters $x_i \in x_i$ ($i = 1, 2, \dots, Q$), which can be described by the notation $\mathbf{x} \equiv (x_1, x_2, \dots, x_Q)$, the signal $U(t; \mathbf{x})$ is recorded during the observation interval $-\frac{\theta}{2} \leq t < \frac{\theta}{2}$ [3]. The dependence of the signal $U(t; \mathbf{x})$ on the parameters \mathbf{x} is dynamic. The values of x_i can be a set of the first natural numbers – this leads to the simplest version of a multidimensional signal

$$U(t; \mathbf{x}) \sim (U_1(t), U_2(t), \dots, U_Q(t)).$$

Let us consider autonomous objects, in whose dynamic equations

$$D_0[U(t; \mathbf{x}); \mathbf{x}] = 0 \quad (1)$$

time t is not explicitly included.

Assuming that the observed changes in the signal are well approximated by the dynamic equation (1), we assume that the intensity of the response of the corresponding dynamic system to the fluctuation disturbance $F(t; x)$ is sufficiently small. According to this, we will assume that internal fluctuations do not take the signal out of the region of the asymptotically stable E-limit trajectory of the dynamic system [3].

If the fluctuations are so small that the representative point of the object during the signal observation interval $(-\Theta/2, \Theta/2)$ practically does not go beyond the boundaries of the circle, then the signal equation takes the form

$$\frac{d\sigma}{dt'} + \sum_{k=1}^{q-1} A_0^{(k)}(t'; x) n_k = F_0(t'; x); \quad (2)$$

$$\frac{dn_m}{dt'} + \sum_{k=1}^{q-1} A_m^{(k)}(t'; x) n_k = F_m(t'; x) \quad (m = 1, 2, \dots, q-1). \quad (3)$$

In this situation, the problem of analyzing the dynamic structure of an object for each value of x after calculating the position of the ω -limit dynamic trajectory $U_0^+(x)$ in the phase space $R_q(x)$ is reduced to estimating the numerical values of the coefficients $A_0^{(k)}, A_m^{(k)}$ ($k, m = 1, 2, \dots, q-1$).

Let us discuss the calculation scheme that allows us to estimate the values of the coefficients $A_0^{(k)}$ in equation (2). Let us rewrite this equation for an arbitrary realization of $U_\gamma(t; x)$ in a more convenient form:

$$\begin{aligned} \frac{d\sigma_\gamma}{dt'}(t' + \tau; x) + \sum_{k=1}^{q-1} A_0^{(k)}(t' + \tau; x) n_{k\gamma}(t' + \tau; x) &= F_{0\gamma}(t' + \tau; x) \\ (\gamma = 1, 2, \dots, \Gamma). \end{aligned} \quad (4)$$

Multiplying (4) by $n_{l\gamma}(t'; x)$, ($l = 1, 2, \dots, q-1$) and introducing the notations

$$\chi_{\Gamma 0l}^{(t'; x)}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t' + \tau; x) \frac{d\sigma_\gamma}{dt'}(t' + \tau; x) \quad (5)$$

$$\eta_{\Gamma kl}^{(t'; x)}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t'; x) n_{k\gamma}(t' + \tau; x), \quad \varphi_{\Gamma 0l}^{(t'; x)}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t'; x) F_{0\gamma}(t' + \tau; x)$$

we get formulas

$$\chi_{\Gamma 0l}^{(t'; x)}(\tau) + \sum_{k=1}^{q-1} A_0^{(k)}(t' + \tau; x) \eta_{\Gamma kl}^{(t'; x)}(\tau) = \varphi_{\Gamma 0l}^{(t'; x)}(\tau) \quad (l = 1, 2, \dots, q-1), \quad (6)$$

connecting sample correlation functions. The stochastic relationship disappears at shifts exceeding τ_0 , between time-shifted values of the fluctuation disturbance F that refer to the same x [4]. The stochastic relationship between the signal U , as well as its projections A and n_l ($l = 1, 2, \dots, q-1$), on the one hand, and the force F or its projections F_m ($m = 0, 1, \dots, q-1$), on the other, disappears when the response precedes the force by a time greater than τ_0 , since the response of the dynamic system to the disturbing force is determined only by the preceding values of the force.

Since

$$\left[\varphi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau) \right]^2 = \frac{1}{\Gamma} \left[n_l(t'; \mathbf{x}) F_0(t' + \tau; \mathbf{x}) \right]^2, \quad (7)$$

the coefficients $A_0^{(k)}(t'; \mathbf{x})$ can be estimated in zero approximation by the values $A_0^k(t'; \tau; \mathbf{x})$, based on the approximate formulas

$$\chi_{0l}^{(t' - \tilde{\mathbf{x}})}(\tau) + \sum_{k=1}^{q-1} A_0^{(k)}(t'; \mathbf{x}; \tau) \eta_{kl}^{(t' - \tau; \mathbf{x})}(\tau) = 0, \quad (8)$$

where τ is taken as the shift, which lies within the interval $\tau_0 \leq \tau < \Theta_m(\mathbf{x})$.

To simplify the situation, let us assume that our a priori ideas about the dynamic structure of the analyzed object allow us to represent the sought coefficients in the form of unambiguous expansions

$$A_0^{(k)}(t'; \mathbf{x}) \equiv \sum_{r=1}^R A_0^{(k, r; \mathbf{x})} v_{0r}(t'; \mathbf{x}) \quad (9)$$

in terms of previously known functions. We note that assumption (9) is, in principle, verifiable already because the estimate of the quantities $A_0^{(k)}(t'; \mathbf{x})$, obtained in the zero approximation, based on formulas (8), does not require a priori information of this kind. Substituting equality (9) into equations (6) and setting in the latter $\tau = \tau_p, \tau_p = p\tau_0 \left(p = 1, 2, \dots, P; P \approx \frac{\Theta_M}{\tau_0} \right)$, we obtain to estimate the coefficients $A_0^{(k, r; \mathbf{x})}$ system of equations:

$$\chi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau_p) + \sum_{k=1}^{q-1} \sum_{r=1}^R A_0^{(k, r; \mathbf{x})} v_{0r}(t' + \tau; \mathbf{x}) \eta_{kl}^{(t'; \mathbf{x})}(\tau_p) = \varphi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau_p), \quad (10)$$

where should I take it $t' = t'_1, t'_1 + P\tau_0, t'_1 + 2P\tau_0, \dots, t'_1 + \mathcal{P}P\tau_0$; $t'_1 + \mathcal{P}P\tau_0 \approx t'_2$; $p = 1, 2, \dots, P$; $kl = 1, 2, \dots, q-1$; $r = 1, 2, \dots, R$.

The functions $v_{0r}(t'; \mathbf{x})$ are assumed to be known, the values $\chi_{\Gamma 0l}^{(t'; \mathbf{x})}(t_p)$ and $\eta_{kl}^{(t'; \mathbf{x})}(\tau_p)$ are determined according to (2) from the signal recording. Now, in the right-hand sides of these equations there are stochastically independent quantities for different p . Indeed, for sufficiently large values of Γ , the fluctuations $\varphi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau)$ can be considered distributed according to the normal law, and, as is easy to verify by direct substitution, their correlation for different p ($p = 1, 2, \dots, P$) missing:

$$\varphi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau') \varphi_{\Gamma 0k}^{(t'; \mathbf{x})}(\tau') = 0 \quad |\tau' - \tau''| \geq \tau_0. \quad (11)$$

Therefore, starting from system (10), we can estimate the coefficients $A_0^{(k, r; \mathbf{x})}$ using the generally accepted method of finding the minimum of the mean square error [5]. It is the coefficients $A_0^{(k, r; \mathbf{x})}$ that are estimated by the values $\mathcal{A}_0^{(k, r; \mathbf{x})}$ that realize the minimum of the functional

$$\Phi \left(\mathcal{A}_0^{(k, r; \mathbf{x})}; k = 1, 2, \dots, q-1; r = 1, 2, \dots, R \right) \equiv$$

$$\equiv \sum_{t', p, l} \frac{\left[\chi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau_p) + \sum_{k=1}^{q-1} \sum_{r=1}^R \mathcal{A}_0^{(k, r; \mathbf{x})} \nu_{0r}(t' + \tau_p; \mathbf{x}) \eta_{\Gamma kl}^{(t'; \mathbf{x})}(\tau_p) \right]^2}{g_{0t'pl}^{(x)}(\mathcal{A}_0^{(k, r; \mathbf{x})}; k=1, 2, \dots, q-1; r=1, 2, \dots, R)}. \quad (12)$$

Included in (12) the expressions

$$g_{0t'pl}^{(x)} \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} \left[\chi_{0t'\gamma}^{(t'; \mathbf{x})}(\tau_p) + \sum_{k=1}^{q-1} \sum_{r=1}^R \mathcal{A}_0^{(k, r; \mathbf{x})} \nu_{0r}(t' + \tau_p; \mathbf{x}) \eta_{kl\gamma}^{(t'; \mathbf{x})}(\tau_p) \right]^2. \quad (13)$$

where

$$\chi_{0t'\gamma}^{(t'; \mathbf{x})}(\tau) \equiv n_{t'\gamma}(t'; \mathbf{x}) \frac{d\sigma_{\gamma}}{dt'}(t' - \tau; \mathbf{x}), \quad \eta_{kl\gamma}^{(t'; \mathbf{x})}(\tau) \equiv n_{t'\gamma}(t'; \mathbf{x}) n_{k\gamma}(t' - \tau; \mathbf{x})$$

estimate the variances of the “fluctuation errors” $\varphi_{(\tau)}^{(t'; \mathbf{x})}(\tau)$. Finding the values of $\widehat{\mathcal{A}}_0^{(k, r; \mathbf{x})}$ directly by equating to zero the partial derivatives with respect to $\mathcal{A}_0^{(k, r; \mathbf{x})}$ of the function $\Phi_0^{(x)}$ taking into account (13) is a rather cumbersome task.

Its solution can also be found by successive linear approximations, which can be formed as follows: let the values $\mathcal{A}_{0\mu}^{(k, r; \mathbf{x})}$ realize the minimum of the μ -th ($\mu=1, 2, \dots$) functional $\Phi_{0\mu}^{(x)}$:

$$\Phi_{0\mu}^{(x)} \equiv \sum_{t', p, l} \frac{1}{g_{0\mu-1}^{(x)}(t', p, l)} \left[\chi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau_p) + \sum_{k=1}^{q-1} \sum_{r=1}^R \mathcal{A}_{0\mu-1}^{(k, r; \mathbf{x})} \nu_{0r}(t' + \tau_p; \mathbf{x}) \eta_{\Gamma kl}^{(t'; \mathbf{x})}(\tau_p) \right]^2, \quad (14)$$

where the values of the error variances $g_{0\mu-1}^{(x)}$ are estimated based on the solution of the previous approximation:

$$g_{0\mu-1}^{(x)}(t', p, l) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} \left[\chi_{0t'\gamma}^{(t'; \mathbf{x})}(\tau_p) + \sum_{k=1}^{q-1} \sum_{r=1}^R \widehat{\mathcal{A}}_{0\mu-1}^{(k, r; \mathbf{x})} \nu_{0r}(t' + \tau_p; \mathbf{x}) \eta_{kl\gamma}^{(t'; \mathbf{x})}(\tau_p) \right]^2. \quad (15)$$

For $\mu=1$ the values obtained from (8), (9) should be substituted into expressions (14), (15); finally, one can simply set $g_{00}^{(x)}(t', p, l) \equiv 1$.

The evaluation of the components $\mathcal{A}_m^{(k)}$ of the coefficient matrix of system (3) is carried out similarly. In this case, instead of the random vector $\chi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau)$ ($l=1, 2, \dots, q-1$), a random matrix $\chi_{\Gamma ml}^{(t'; \mathbf{x})}(\tau)$ is introduced, where

$$\chi_{\Gamma ml}^{(t'; \mathbf{x})}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{t'\gamma}(t'; \mathbf{x}) \frac{dn_{m\gamma}}{dt'}(t' + \tau; \mathbf{x}) \quad (l, m=1, 2, \dots, q-1), \quad (16)$$

and instead of the vector of fluctuation errors $\varphi_{\Gamma 0l}^{(t'; \mathbf{x})}(\tau)$ ($l=1, 2, \dots, q-1$) the matrix $\varphi_{\Gamma ml}^{(t'; \mathbf{x})}(\tau)$ is introduced, in which

$$\varphi_{\Gamma ml}^{(t'; \mathbf{x})}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{t'\gamma}(t'; \mathbf{x}) F_{m\gamma}(t' + \tau; \mathbf{x}). \quad (17)$$

Conclusions

Thus, based on a sufficiently long and detailed recording of the signal $U(t; \mathbf{x})$ using cumbersome numerical processing, it is possible to estimate the dynamic characteristics of a weakly fluctuating unregulated object (of a fairly general type) near its ω -limit trajectory that does not degenerate into a rest point.

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