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ADDITIVE VS ADDITIVE–MULTIPLICATIVE MODELS FOR NONINVASIVE FETAL ELECTROCARDIOGRAPHY: CASCADE EXTRACTION METHODS AND NOISE ROBUSTNESS

Transcutaneous fetal electrocardiogram (ECG) recording is considered a task of separating components in a mixed abdominal signal containing maternal ECG (MECG), fetal ECG (FECG), and noise. The article systematically compares two formulations – additive (linear superposition) and additive-multiplicative – in the time and frequency domains, including parameter identification and component separation procedures, as well as the assessment of resistance to typical interference and recording artifacts. It is shown that for most practical scenarios, an additive model with a processing cascade is adequate: zero-phase band filtering and amplitude normalization; adaptive suppression of the maternal component (LMS/RLS in the presence of a chest reference or ICA for multichannel recordings); local isolation of informative details using wavelet and multiresolution analysis (CWT/DWT, MRA); final refinement by weighted least squares. When cross-terms cause combinatorial frequencies, it is advisable to switch to an additive-multiplicative model, detecting nonlinearities through bispectral analysis and selectively suppressing them in the time-frequency representation. If spectral power estimates are available, a Wiener filter is effective. Quality is assessed by SNR, relative reproduction error (RRE), and QRS detection metrics (sensitivity, positive predictive value). The proposed cascade scheme minimizes overfiltering, preserves QRS morphology, and increases noise immunity to isoline drift, network interference, motion/contact, and myogenic artifacts.

Key words: *FECG, MECG, additive model, additive-multiplicative model, LMS, ICA, DWT/CWT, MRA, Wiener filter, bispectrum, noise, baseline drift, motion artifacts.*

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АДИТИВНА ТА АДДИТИВНО-МУЛЬТИПЛІКАТИВНА МОДЕЛІ ДЛЯ НЕІНВАЗИВНОЇ ЕЛЕКТРОКАРДІОГРАФІЇ ПЛОДА: МЕТОДИ КАСКАДНОЇ ЕКСТРАКЦІЇ ТА ЗАВАДОСТІЙКІСТЬ

Запис черезшкірної електрокардіограми (ЕКГ) плода розглядається як завдання розділення компонентів у змішаному абдомінальному сигналі, що містить ЕКГ матері (МЕКГ), ЕКГ плода (ФЕКГ) і шуму. У статті систематично порівнюються два способи подання таких сигналів – як адитивна (лінійна суперпозиція) та адитивно-мультиплікативна суміш – у часовій та частотній областях, включаючи процедури ідентифікації параметрів та розділення компонентів, а також оцінку стійкості до типових перешкод та артефактів запису. Показано, що для більшості практичних сценаріїв адекватною є адитивна модель з каскадом обробки: фільтрація нульової фази та нормалізація амплітуди; адаптивне придушення материнської складової (LMS/RLS за наявності опорного сигналу з грудної клітки або ICA для багатоканальних записів); локальна ізоляція інформативних ознак за допомогою вейвлет-аналізу та аналізу з кількома роздільними здатностями (CWT/DWT, MRA); остаточне уточ-

нення за допомогою зважених найменших квадратів. Коли перехресні члени викликають комбінаторні частоти, доцільно перейти до адитивно-мультиплікативної моделі, виявляючи нелінійності за допомогою біспектрального аналізу та вибірково придушуючи їх у часово-частотному представленні. Якщо доступні оцінки спектральної густини потужності, ефективним буде застосування фільтра Вінера. Якість оцінюється за співвідношенням сигнал/шум (SNR), відносною помилкою відтворення (RRE) та метриками виявлення комплексу QRS (чутливість, позитивна прогностична цінність). Запропонована каскадна схема мінімізує надмірну фільтрацію, зберігає морфологію комплексу QRS та підвищує завадостійкість до дрейфу ізоліній, мережевих перешкод, руху/контакту та міогенних артефактів.

Ключові слова: *ФЕКГ, МЕКГ, адитивна модель, адитивно-мультиплікативна модель, LMS, ICA, DWT/CWT, MRA, фільтр Вінера, біспектр, шум, дрейф ізолінії, артефакти руху.*

Introduction

Extraction of the fetal electrocardiogram (FECG) is of primary importance in the field of monitoring the condition of the fetus and prenatal diagnosis, as it makes it possible to assess the state of the cardiovascular system of the fetus in a non-invasive way. The problem is that (FECG) signals are quite strongly hidden in the maternal electrocardiographic (MECG) signals, which makes the problem of their extraction one of the most difficult in the field of biomedical engineering. Successful extraction of the fetal signal is not only fundamental, but also critically important, as it allows for the early detection of congenital heart defects, arrhythmias and fetal distress, which often indicate the presence of hypoxia or other life-threatening conditions. The FECG signal, unlike the adult ECG signal, is subject to a wide range of external and internal factors that affect its clarity and amplitude.

Continuous monitoring of fetal condition is a decisive factor in reducing perinatal mortality. According to WHO estimates, there are approximately 2.6 million stillbirths each year, a significant proportion of which could be prevented through timely and accurate monitoring [1]. Non-invasive FECG signal monitoring provides continuous, real-time assessment of fetal well-being, aligning with the WHO’s global mission [2] and offering a logical response to the limitations of Doppler and intermittent auscultation, which provide only episodic measurements and may miss subtle signs of distress [3]. The WHO’s goal of reducing stillbirths to <12 per 1,000 births by 2030 underscores the need for widespread adoption of advanced monitoring techniques, including FECG signal [4]. The advantages of this approach include continuity, non-invasiveness, and better detection of subtle abnormalities in fetal cardiac activity (in particular through analysis of QRS complexes and heart rate variability, HRV) [5], [6], as well as the potential to reduce inequalities in access to quality prenatal care [7], which together contribute to improved perinatal outcomes [8].

Formulation of the research objective

The objective of the article is to justify the choice of the mathematical describing the fetal ECG signal and methods for its processing based on a study of the main advantages and disadvantages of the classes of additive and additive-multiplicative models.

Additive model: basic formulation

The observed abdominal signal is presented as a linear superposition of MEX, attenuated FEX, and noise:

$$x_{abd}(t) = m(t) + \alpha(t)f(t) + n(t). \quad (1)$$

Here, m(t) is MECG, f(t) is FECG, α(t) is the time-varying attenuation/scaling coefficient, and n(t) is noise. To reflect frequency-dependent attenuation and phase shifts introduced by biological tissues, time-varying transfer functions $H_m(t)$ and $H_f(t)$) are introduced with the convolution formulation:

$$(t) = \int_{-\infty}^{\infty} H_m(\tau,t)\,m(t-\tau)\,\mathrm{d}\tau + a_f \int_{-\infty}^{\infty} H_f(\tau,t)\,f(t-\tau)\,\mathrm{d}\tau + n(t). \quad (2)$$

In the frequency domain

$$S(f) = H_m(f)M(f) + \alpha_f H_f(f)F(f) + N(f), \quad (3)$$

where $M(f)$, $F(f)$, $N(f)$ are the Fourier transforms of MECG, FECG, and noise, respectively. Typically, MECG energy is concentrated in the range of 0.5–2 Hz, while FECG is in the range of 1–5 Hz (110–160 beats per minute), which is the basis for band selection.

Noise n(t) is often modeled as a Gaussian process with zero mean:

$$n(t) \sim N(0, \sigma^2). \quad (4)$$

For impulse noise, the Laplace model is used:

$$p(n) = \frac{1}{2b}\exp\left(-\frac{|n|}{b}\right), \quad (5)$$

and for non-stationary conditions, time-varying variance is used:

$$n(t) \sim N(0, \sigma^2(t)). \tag{6}$$

This formulation is consistent with real recordings, which contain 50/60 Hz line noise, motion artifacts, baseline drift, electrode noise, and myogenic noise.

Additive-multiplicative model: accounting for interactions

If there are noticeable interactions between components, a quadratic term is added to the additive terms:

$$s_{obs}(t) = A_m s_m(t) + A_f s_f(t) + B_m S_m(t) s_f(t) + n(t). \tag{7}$$

The multiplicative term generates sum-difference frequencies. Representing $s_m(t)$ and $s_{mf}(t)$ as finite Fourier series:

$$s_m(t) = \sum_{k=1}^{K} A_k \cos(\omega_k t + \phi_k), \tag{8}$$

$$s_f(t) = \sum_{l=1}^{L} B_l \cos(\omega_l t + \psi_l), \tag{9}$$

the product $s_m(t)\, s_f(t)$ decomposes into cosines with frequencies $\phi_k + \psi_l$:

$$s_m(t) s_f(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \frac{A_k B_l}{2} \cos\big((\omega_k + \omega_C)t + (\phi_k + \psi_l)\big) + \cos\big((\omega_k - \omega_C)t + (\phi_k - \psi_l)\big). \tag{10}$$

In the frequency domain, the convolution of spectra yields peaks at $\omega_k + \omega_l$, which complicates pure band separation. The spectral power density (PSD) of the observed signal has the form:

$$s_{m\cdot f}(\omega) = B_m \sum_{k=1}^{K} \sum_{l=d}^{L} \frac{A_k B_l}{2} [\delta(\omega - (\omega_k + \omega_l)) + \delta(\omega - (\omega_k - \omega_l))]. \tag{11}$$

Identification of parameters and separation of components

Additive model

Adaptive filtering (LMS). Using the reference estimate MEKS $\widehat{m}(t)$, the error $e(t)$ is minimized:

$$\mathrm{e}(t) = S(t) - \widehat{m}(t) = \alpha_f f(t) + n(t), \tag{12}$$

updating the weights $w(t)$:

$$w(t+1) = w(t) + \mu e(t) S(t). \tag{13}$$

The residual $S(t)$ contains FECS and noise.

Independent component analysis (ICA). For multichannel recordings

$$S(t) = A \cdot X(t), \tag{14}$$

$$X(t) = W \cdot S(t), \tag{15}$$

where $X(t)$ are independent sources (MECS, FECS). The goal is to find W that maximizes the independence of the components.

Time-frequency analysis. Continuous wavelet transform:

$$C(a,b) = \int_{-\infty}^{\infty} S(t) \psi^* \left(\frac{t-b}{a} \right) \mathrm{d}t, \tag{16}$$

and discrete wavelet decomposition DWT performs multilevel decomposition

$$S(t) \to \{A_1, D_1\}, \{A_2, D_2\}, \ldots, \{A_n, D_n\}. \tag{17}$$

At lower levels (low frequencies), MECG dominates, while at higher levels (1–5 Hz), FECG dominates. Multiresolution analysis (MRA)

$$S(t) = A_n(t) + \sum_{i=1}^{n} D_i(t), \tag{18}$$

allows us to reconstruct FECS, preserving only details from the k-th level:

$$f(t) \approx \sum_{i=k}^{n} D_i(t). \tag{19}$$

Wavelet denoising. Thresholding of coefficients

$$\tilde{C}(a,b) = \begin{cases} C(a,b) & if\ |C(a,b)| > \lambda \\ 0 & else \end{cases}, \tag{20}$$

suppresses impulse/high-frequency noise while preserving the QRS morphology. Wavelet families suitable for ECG are selected based on their similarity to the QRS complex and the "time-frequency" compromise.

Working with non-stationarity. Adaptive filters (LMS/RLS) and Kalman filters are used for motion and drift artifacts; baseline correction can be performed by HF filtering or polynomial fitting.

Additive-multiplicative model

The presence of cross terms ($\omega_k + \omega_l$) is detected using higher spectral moments – bispectrum analysis. To isolate the FEC in this case, two steps are helpful:

Fundamental separation of the additive part (DWT/MRA, LMS, or ICA),

Suppression of cross-terms by selective processing in the frequency or time-frequency domain (masking of peaks $\omega_k + \omega_l$).

Additionally, consider a Wiener filter in the frequency domain to minimize the root mean square error between the estimated and desired FECS:

$$H_f(\omega) = \frac{P_{s_f}(\omega)}{P_{s_f}(\omega) + A_m^2 P_{s_m}(\omega) + B_m^2\left(P_{s_m}(\omega)\cdot P_{s_f}(\omega)\right) + P_N(\omega)}. \quad (21)$$

The time-frequency representation (CWT) with the ψ kernel allows local processing of non-stationary cross-effects:

$$W_{obs}(a,b) = \int_{-\infty}^{+\infty} s_{obs}(t)\,\psi\left(\frac{t-b}{a}\right)\mathrm{d}t. \quad (22)$$

Sensitivity to noise and artifacts

The effectiveness of ECG extraction from abdominal recordings is determined by the processing cascade's ability to suppress typical interferences, including low-frequency baseline drift, high-frequency noise, residual network components, motion/contact artifacts, and myogenic noise, as well as combination frequencies arising from additive-multiplicative interactions. The baseline and HF noise are suppressed by bandpass zero-phase filtering, after which the amplitude is stabilized by normalization; non-stationary noise is compensated by adaptive subtraction of the LMS mother component, and local emissions reduce the impact through weighted least squares. If necessary, bispectral analysis is used to detect nonlinear cross-correlations.

Bandpass filtering isolates the informative range of the fECG and simultaneously cuts off drift (<1 Hz) and HF noise. The ideal prototype is described by:

$$H_{bp}(f) = \begin{cases} 1, & f_L \le f \le f_U \\ 0, & \textit{інакше} \end{cases}, \quad (23)$$

and structurally use a zero-phase Butterworth implementation equivalent to the product of two "slices":

$$H_{bp}(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_L}\right)^{2n}}}\cdot\frac{1}{\sqrt{1+\left(\frac{f}{f_U}\right)^{2n}}}. \quad (24)$$

Frequency composition control is performed via the Fourier transform:

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\mathrm{d}t, \quad (25)$$

and time distortions are tracked by group delay:

$$\tau_g(f) = -\frac{\mathrm{d}\phi(f)}{\mathrm{d}f}, \quad (26)$$

which zero-phase filtering effectively eliminates while preserving the QRS morphology.

To prevent subsequent stages from being sensitive to amplitude variations, single or robust normalization is applied:

$$A_{\max} = \max(|x_{bp}(t)|), \quad (27)$$

$$x_{norm}(t) = \frac{x_{bp}(t)}{A_{\max}}, \quad (28)$$

$$A_{robust} = P_{95}(|x_{bp}(t)|), \quad (29)$$

where P_{95} – 95th percentile. Such scale stabilization accelerates and stabilizes the convergence of adaptive filters, simplifying thresholding during peak detection.

Adaptive suppression of the mother component is performed by an LMS filter with the chest ECG as the reference channel. The output of the filter is of order N:

$$y(\mathrm{i}) = \sum_{j=0}^{N-1} \omega_j \cdot x(\mathrm{i} - j). \quad (30)$$

The error $e(i)$ at each moment in time i is defined as:

$$e(i) = d(i) - y(i), \quad (31)$$

minimizes the root mean square error:

$$J = E[e(i)^2] = E[(d(i) - y(i'))]. \quad (32)$$

To do this, the LMS algorithm updates the filter weights in the direction opposite to the gradient J with respect to ω:

$$\omega_{i+1} = \omega_i + 2\mu e(i)x(i). \quad (33)$$

Small μincreases resistance to noise and motion artifacts, while large values can cause oscillations and loss of convergence.

In the context of FECG signal extraction, least squares optimization serves as a final refinement tool based on the results of previous filtering steps.

The goal of least squares optimization is to minimize the quadratic error between the observed signal $y(i)$ (after adaptive filtering) and the predicted fetal ECG $\hat{f}(i)$. This can be formulated as a loss function J:

$$J\left(\hat{f}(t)\right) = \sum_{i=1}^{N} \left(y(i) - \hat{f}(i)\right)^2. \quad (34)$$

The least squares method finds the vector β that minimizes the sum of the squares of the residuals $e^T e$:

$$\beta = (X^T X)^{-1} X^T y, \quad (35)$$

And to reduce the impact of artifacts, a weighted form is used:

$$J_{WLC}\left(\hat{f}(t)\right) = \sum_{i=1}^{N} \omega_i \left(y(i) - \hat{f}(i)\right)^2, \quad (36)$$

$$X_\omega = W^{\frac{1}{2}} X,\ y_\omega = W^{\frac{1}{2}} y, \quad (37)$$

$$\beta_{WLC} = \left(X_w^T X_\omega\right)^{-1} X_\omega^T y_\omega. \quad (38)$$

Local emissions are identified and corrected according to the interquartile range rule:

$$IQR = Q3 - Q1. \quad (39)$$

In the presence of combinatorial components (sum/difference frequencies) resulting from additive-multiplicative interactions, third-order bispectral analysis is functional:

$$B(\omega_1, \omega_2) = E[s(\omega_1)s(\omega_2)s \cdot (\omega_1 + \omega_2)], \quad (40)$$

which localizes phase-dependent interactions for subsequent selective suppression (e.g., masking in the time-frequency representation). The ratio of the spectra controls the basic frequency balance of the mixture:

$$s_{obs}(\omega) = s_m(\omega) + s_f(\omega) + N(\omega). \quad (41)$$

Qualitative noise immunity is evaluated using energy and time metrics, including the signal-to-noise ratio (as the ratio of the root mean square powers of the estimated FECG and residual noise after subtraction), relative reproduction error (RRE), and QRS detection indicators (sensitivity and positive predictive value). The working sequence is as follows. First, zero-phase bandpass filtering is performed to suppress drift and high-frequency noise. Next, unit or robust normalization is applied to stabilize the signal scale. After that, adaptive subtraction is used according to the LMS algorithm with a correctly selected μ parameter to suppress the mother component. The next step is to refine the waveform using the least squares or weighted least squares method to reduce the influence of locally noisy areas. If there are signs of additive-multiplicative interactions, bispectral detection of combination frequencies and selective suppression of the corresponding components are additionally performed. This cascaded organization minimizes overfiltering, preserves the morphology of the QRS complex, and reduces sensitivity to typical noise and artifacts.

Conclusions

In conclusion, the comparison of additive and additive-multiplicative approaches to the problem of isolating FECG from abdominal recordings revealed that linear superposition is a sufficiently accurate model for most practical scenarios, providing consistent processing in both the time and frequency domains, based on the frequency separation of MECG and FECG. At the same time, the presence of cross-interactions between the maternal and fetal components, which generate combinatorial (sum-difference) frequencies, requires a transition to an additive-multiplicative model, since pure band separation becomes insufficient. For such cases, it is advisable to use bispectral analysis to identify nonlinear terms and their subsequent selective suppression in the time-frequency representation. If spectral powers are estimated, a Wiener filter is also helpful as a solution, minimizing the root mean square error.

The effectiveness of cascade processing, which begins with zero-phase bandpass filtering (to suppress baseline drift and high-frequency noise without distorting the QRS morphology) and amplitude normalization (conventional or robust), has been experimentally demonstrated. Further adaptive subtraction of the maternal component (LMS/RLS in the presence of a chest reference or ICA for multichannel recordings) in combination with wavelet and multiresolution analysis (CWT/DWT, MRA) provides local isolation of informative FECG details. Final refinement using the least squares method, particularly in a weighted setting, reduces the impact of local emissions and contact/motion artifacts. The proposed sequence minimizes the risk of over-filtering, increases resistance to typical interference (including network components and myogenic noise), and preserves diagnostically significant morphology.

Quality assessment should be performed comprehensively, considering the signal-to-noise ratio, relative reproduction error, and QRS detection metrics (sensitivity and positive predictive value). The practical recommendation is that the additive model, together with the described processing cascade, should be considered the standard choice in routine monitoring conditions. The additive-multiplicative model should be used when there are convincing signs of nonlinear cross-interactions, confirmed by the bispectrum. Prospects for further research include adapting filtering parameters in online mode using Bayesian/Kalman estimators and standardizing validation protocols on open datasets with well-defined noise and artifact scenarios.

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