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MATHEMATICAL MODEL OF BLOCK PROCESS PLANNING IN SYSTEMS OF ALLOCATION OF TASK BETWEEN PEOPLE AND COLLABORATIVE ROBOTS IN THE FRAMEWORK OF INDUSTRIES 5.0

This article considers the current problem of task allocation between humans and collaborative robots in the context of Industry 5.0 using block process planning. The main focus is on analyzing the interaction between operators and automated systems operating in a shared production environment. The main goal is to ensure harmonious cooperation between humans and robots by optimizing task allocation, taking into account a number of important factors, such as time and resource constraints, the complexity of the operations performed, the level of autonomy of robotic systems, and the priority of performing different stages of production. As part of the study, a mathematical model is proposed that includes cost and benefit functions that allow assessing x1the effectiveness of planning. The model also contains numerous time and resource constraints that are critical to maintaining the productivity, safety, and flexibility of modern production systems. To verify its operability, software in Python was developed that allows not only to automatically carry out the planning process, but also to evaluate the overall effectiveness of the proposed task allocation strategies. The conducted experimental studies have shown that the success of planning depends to a large extent on the balance of time and resource parameters. The conducted experiments have shown that the success of planning depends on the balance of time and resource parameters: at values and all constraints are met, and the cost function fluctuates within 30–80. In contrast, in the case of insufficient resources, the system exhibits increased sensitivity, which makes the performance of some tasks impossible or inefficient. The results obtained confirm that the developed model is resistant to parameter changes and provides optimal task distribution in most production scenarios. Prospects for further research include extending the model for dynamic environments, integrating machine learning algorithms for forecasting, and improving the adaptive planning process.

Key words: *block process planning, task allocation, collaborative robots, Industry 5.0, optimization, cost function, resource constraints.*

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МАТЕМАТИЧНА МОДЕЛЬ БЛОЧНОГО ПРОЦЕСНОГО ПЛАНУВАННЯ В СИСТЕМАХ АЛОКАЦІЇ ЗАВДАНЬ МІЖ ЛЮДЬМИ ТА КАЛАБОРАТІВНИМИ РОБОТАМИ В РАМКАХ ІНДУСТРІЙ 5.0

У даній статті розглянуто актуальну проблему алокації завдань між людьми та колаборативними роботами в умовах Індустрії 5.0 з використанням блочного процесного планування. Основна увага приділяється аналізу взаємодії між операторами та автоматизованими системами, що працюють у спільному виробничому середовищі. Основною метою є забезпечення гармонійної співпраці між людьми та роботами шляхом оптимізації розподілу завдань з урахуванням низки важливих факторів, таких як часові та ресурсні обмеження, складність виконуваних операцій, рівень автономності роботизованих систем, а також пріоритетність виконання різних етапів виробництва. У рамках дослідження запропоновано математичну модель, яка включає функції вартості та вигідності, що дозволяють оцінити ефективність планування. Модель також містить численні обмеження на час і ресурси, що є критично важливими для підтримки продуктивності, безпеки та гнучкості сучасних виробничих систем. Для перевірки її працездатності було розроблено програмне забезпечення на мові Python, яке дозволяє не лише автоматично здійснювати процес планування, але й оцінювати загальну ефективність запропонованих стратегій розподілу завдань. Проведені експериментальні дослідження продемонстрували, що успішність планування значною мірою залежить від збалансованості часових і ресурсних параметрів. Проведені експерименти показали, що успішність планування залежить від збалансованості часових і ресурсних параметрів: при значеннях $T_{max} \geq 5$ і $R_{max} \geq 7$ всі обмеження виконуються, а функція вартості коливається в межах 30–80. Натомість у разі недостатності ресурсів система виявляє підвищену чутливість, що робить виконання деяких завдань неможливим або неефективним. Отримані результати підтверджують, що розроблена модель є стійкою до змін параметрів і забезпечує оптимальний розподіл завдань у більшості виробничих сценаріїв. Перспективи подальших досліджень включають розширення моделі для динамічних середовищ, інтеграцію алгоритмів машинного навчання для прогнозування та вдосконалення процесу адаптивного планування.

Ключові слова: *блочне процесне планування, алокація завдань, колаборативні роботи, Індустрія 5.0, оптимізація, функція вартості, ресурсні обмеження.*

Statement of the problem

The modern development of industry, based on the concept of Industry 5.0, focuses on harmonious interaction between people and robots, in particular collaborative ones. The main task of such systems is not only to automate production processes, but also to ensure flexible distribution of tasks between human operators and robots to achieve maximum efficiency, productivity and safety. However, this approach creates a number of problems related to synchronization of actions, optimal use of resources and real-time task planning. Existing algorithms often do not take into account the dynamics of the environment, the psychological and physical characteristics of people, as well as the limitations that arise when robots work in close contact with personnel. In particular, the problem of block process planning, which involves the distribution of tasks within certain blocks of the production cycle, is a difficult task from the point of view of developing a universal mathematical model. Insufficient attention is paid to methods that simultaneously take into account the priority of tasks, the need for adaptability to environmental changes and the need to ensure the continuity of production processes. Such approaches are critically important for achieving the main goal of Industry 5.0 – creating personalized, human-centered and sustainable production. Thus, the development of a mathematical model of block process planning, which will allow integrating the human factor and the capabilities of collaborative robots, is relevant and has significant scientific and practical interest.

Analysis of recent research and publications

Lima, RKD, Heckler, WF et al. proposed a systematic mapping and taxonomy for integrating collaborative learning and advanced technologies in the context of Industry 5.0. The main advantage of their approach is that it provides a general methodology that can be useful for building flexible learning systems for robots and humans [1]. However, from the perspective of the task of developing a mathematical model of block process planning, the proposed solutions do not take into account specific constraints on task execution time and resource optimization in real time. Their approach focuses more on interaction and learning than on efficient planning and allocation. Zafar, MH, Langås in their research have investigated the synergies between collaborative robots, digital twins and the complement of technologies for smart manufacturing. The advantage of this approach is the deep integration of digital twins, which allows for the simulation of scenarios and the testing of different planning strategies [2]. However, from the perspective of the block process planning problem, their results do not provide a detailed methodology for determining the optimal parameters of tasks or resources. The main emphasis is on technological integration, which is not always effective for local optimization of production processes. Rahman, MM, Khatun consider the evolutionary roles of collaborative robots within the framework of Industry 5.0. The strength of their approach lies in the analysis of promising capabilities of new generation robots, such as human sensitivity and adaptation to environmental changes [3]. However, a disadvantage is the lack of specific mathematical models for planning and task allocation. Their work is useful for developing general concepts, but does not offer effective algorithms for optimizing block planning. In the article Raffik, R., Sathya [4] consider improving human-robot collaboration using collaborative robots. The advantage is the focus on human factors and building safe environments for collaboration [5]. However, their review approach does not provide mathematical models or algorithms that can be applied to task allocation systems. Thus, the use of their results is limited to scheduling optimization problems. Authors Oladeinde, A. H., & Ojo, O. O. focus on production planning and control in the context of Industry 5.0. An important advantage is their emphasis on optimizing production resource management and long-term planning [5]. However, their approaches are poorly suited for dynamic block planning tasks, since the main focus is on strategic rather than operational aspects. In the article Kumar, S. S., Kumar, S. R., & Ramesh, G. investigated the evolution from Industry 4.0 to 5.0 through the enrichment of production processes with technology and human-robot interaction. Their approach includes key principles of technical improvement and automation integration, which is useful for the overall development of concepts [6]. However, a drawback is the lack of attention to operational planning models, which does not allow using their results to solve problems in block process planning systems.

Formulation of the research objective

The aim of the research is to develop a mathematical model of block process planning for optimal task allocation between humans and collaborative robots in the dynamic production environment of Industry 5.0. The model should take into account the features of human-robot interaction, task prioritization, adaptability to environmental changes, and requirements for safety and efficiency. This will ensure the harmonious integration of the human factor and robotic technologies into production processes, increase the productivity and flexibility of systems. The research is also aimed at creating algorithmic support for implementing the model in a real production environment.

Development of a mathematical model of block process planning in task allocation systems between people and collaborative robots

In modern conditions, industry is increasingly focused on cooperation between humans and collaborative robots, which allows achieving high efficiency and adaptability of production [7]. An important aspect of this process is the development of planning models that take into account the interaction of humans and robots, ensuring the optimal distribution of tasks according to their capabilities and the current production situation. Block process planning becomes necessary for structured and flexible management of workflows, where the precise definition of the scope, priorities and sequence of tasks plays a key role. In this context, task allocation distributes tasks between humans and robots by optimizing according to their functional characteristics and process requirements.

For block process planning in task allocation systems between humans and collaborative robots, it is necessary to develop mathematical models with parameters that take into account the set of tasks, resources for their execution, and constraints [8].

Let us denote within the framework of these studies the problem model (T) as a set of problems (t_n), which can be represented by the following expression:

$$T = \{t_1, t_2, \dots, t_n\}, \tag{1}$$

where: t_n is a task that has characteristics describing its requirements and constraints. These characteristics are necessary for optimal allocation among resources.

Then the problem t_n can be posed as a set of the following characteristics:

$$t_n = \langle d_n, c_n, p_n, r_n \rangle, \tag{2}$$

where: d_n – time required to complete the task t_n. Used to calculate the total time required to complete all tasks; c_n – a measure of the complexity of the task t_n. Complex tasks are typically assigned to human resources who can perform intellectual or creative tasks, while robots perform mechanical tasks; p_n – task priority t_n, which determines the order in which tasks are performed. This is important for tasks that require urgent execution; r_n – the resources required to complete the task t_n, which determines the use of collaborative robots and humans.

To model the assignment of the task, a variable is introduced x_{ij}, where $x_{ij} = 1$ means that the task t_n assigned to the resource j (a job or a person), otherwise $x_{ij} = 0$.

To optimize time or financial resources, it is proposed use the general cost function:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_n \cdot d_n \cdot x_{ij}. \tag{3}$$

Where c_i – the cost of execution, which depends on the type of task (t_n) and the resource (r_n).

The model should also take into account the total time to complete all tasks:

$$T_{total} = \sum_{i}^{n} d_n \cdot x_{ij}. \quad (4)$$

This allows you to minimize execution time while maintaining overall efficiency.

Robots (R) and people (H) are represented by sets of resources:

$$R = \{r_1, r_2, \ldots, r_m\}; \quad (5)$$

$$H = \{h_1, h_2, \ldots, h_k\}. \quad (6)$$

Each resource has accessibility settings a_i and cost of use c_i

The allocation problem is optimized using the cost function Z, which is minimized by a chosen criterion, for example, time or cost:

$$Z = \sum_{i=1}^{n} c_i x_{ij}. \quad (7)$$

Where: $x_{ij} = 1$, if the task t_i assigned to the resource r_i, otherwise $x_{ij} = 0$.

We introduce time and resource constraints to ensure efficient and rational allocation of tasks between humans and robots. Time constraints ensure that all tasks are completed within the allowable time frame, preventing delays and schedule violations. Resource constraints ensure optimal use of available resources, preventing them from being overloaded or idle. This allows for a balance between productivity and efficiency in the operation of hybrid systems that combine human and robotic potential. Time constraints can be represented as follows:

$$\sum_{i}^{n} d_i \cdot x_{ij} \le T_{\max}, \quad \forall j \quad (8)$$

$$\sum_{j=1}^{m} x_{ij} = 1, \ \forall i. \quad (9)$$

Where: $T_{\max}$ – the maximum available time for each resource to ensure even time distribution.

In order to increase efficiency It is proposed to use a benefit function that assesses the feasibility of performing a certain task by a certain resource, as can be represented as follows:

$$B(x_{ij}) = \alpha \cdot \text{efficiency} + \beta \cdot \text{flexibility} + \gamma \cdot \text{safety} \quad (10)$$

Where: α, β, γ – weighting factors.

The proposed mathematical expressions 1–10 make it possible, in the opinion of the authors, to develop an allocation system that takes into account productivity, safety, and adaptability thanks to a block approach to task management of a robotic manipulator in Industry 5.0.

Development of a program to calculate the total cost function for the proposed distribution of tasks among resources

The Python language was chosen for the development of the block process planning program due to its high accessibility, simplicity of syntax, and wide range of libraries supporting mathematical calculations and optimization [9]. Python offers tools such as NumPy, SciPy, and Pandas, which provide efficient work with data, as well as integration with specialized task distribution algorithms. The flexibility of the language allows for rapid prototype development and integration of models with machine learning or analytics systems. In addition, Python is cross-platform, which ensures its use in various production environments and automation systems.

Based on the developed mathematical models (1–10) of block process planning in task allocation systems between humans and robots, a program has been created to check the correctness of the planning. The program receives input data from the user, including parameters of tasks, resources and constraints, calculates cost and benefit functions and outputs the results.

We present a description of the software implementations of the developed mathematical models in the form of program code functions.

```
def calculate_cost ( tasks, resources, assignments) :
cost = 0
for i , task in enumerate(tasks):
for j, assigned in enumerate(assignments [ i ]):
if assigned == 1: # Task intended resource
cost += task ['complexity'] * task ['time']
return cost
```

calculate_cost function is designed to calculate the overall cost function of allocating tasks to resources. It goes through each task and checks whether it is assigned to a specific resource, and if so, adds the product of the task's complexity and time to the total cost. This allows you to evaluate the efficiency of the scheduling in terms of resource utilization.

```
def validate_constraints (tasks, resources, assignments, t_max):
   for i, task in enumerate (tasks):
      total_time = sum (assignments [i][j] * task [‘time’] for j in range (len (resources)))
      if total_time > t_max:
         return False , “Task timed out.”
   for j, resource in enumerate (resources):
      total_resources = sum (assignments [i][j] * tasks [i][‘resources’] for i in range
(len (tasks)))
      if total_resources > resource [‘max_resource’]:
         return False , “Available resource exceeded.”
   return True , «All constraints are met.»
```

validate_constraints function checks that the time and resource constraints for task allocation are met. It calculates the total execution time for each task and the resource utilization for each resource, comparing them to the specified limits. If any constraint is violated, the function returns an appropriate error message; otherwise, it confirms that the schedule is correct.

```
valid, message = validate_constraints (tasks, resources, assignments, t_max)
   print (message)
# Calculation of the cost function
   if valid:
      cost = calculate_cost (tasks, resources, assignments)
      print (f″ Total cost function: {cost}″)
   else :
      print («The correctness of the planning has not been confirmed.»)
```

This code checks the correctness of the task scheduling using the validate_constraints function , printing a message about the result of the check. If the constraints are met, the total cost function is calculated using calculate_cost , and the result is printed to the screen. If the constraints are violated, the scheduling is considered incorrect, and the user receives an appropriate message.

Let's conduct a series of experiments to show at what values of time and resource constraints for task allocation the following result will be obtained: all constraints are met and constraints are not met. The results of successful planning (all constraints are met) are given in Table 1, and for unsuccessful planning (constraints are not met) in Table 2.

Table 1

Results of successful planning (all constraints met)

Max Time Constraint (T_max)	Max Resources Constraint (R_max)	Valid	Cost
4	9	True	34
5	8	True	38
6	7	True	44
6	8	True	40
6	9	True	62
7	6	True	50
7	7	True	56
7	8	True	32

Table 2

Results of failed planning (constraint not met)

Max Time Constraint (T_max)	Max Resources Constraint (R_max)	Valid	Cost
4	7	False	None
5	6	False	None

For convenience of analysis Let us present the obtained experimental data from Tables 1 and 2 in the form of a combined graph, which is presented in Figure 1.

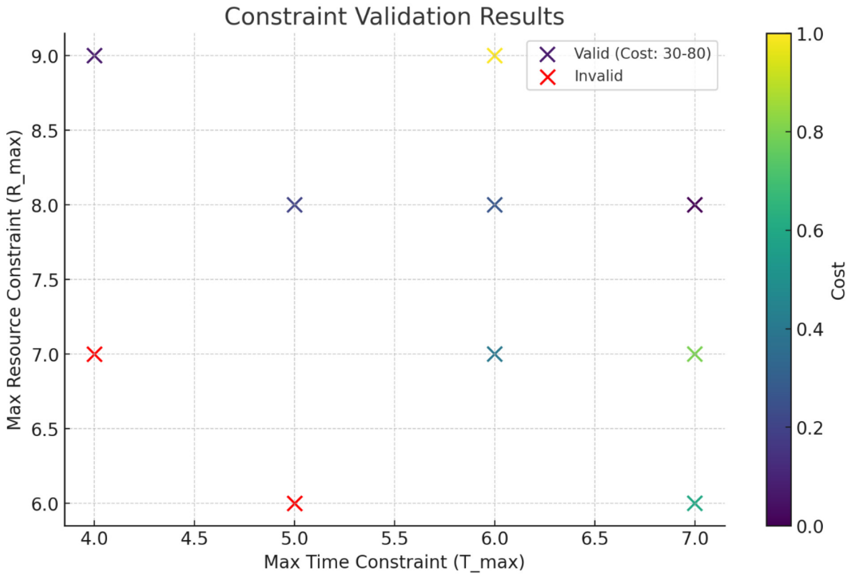

Fig. 1. Combined graph of the obtained experimental results

Figure 1 shows successful planning (total cost function in the range 30–80) and unsuccessful cases where the constraint is not met. Blue represents the cost range and red indicates non-compliance with the constraints.

The results of the experiments showed that the success of task planning depends on the correct combination of time and resource constraints. The first table shows cases when all constraints are met, and the total cost function is within 30–80. At values of $T_{max} \geq 5$ and $R_{max} \geq 7$, the system is able to ensure correct task distribution, which indicates the need for higher resource constraints for complex tasks. In contrast, the second table records violation of constraints at lower values of $T_{max} \leq 4$ and $R_{max} \leq 6$, which led to the impossibility of correct planning. Data analysis shows that the main factor is the balance between available time and resources, since their insufficiency leads to the impossibility of performing complex tasks. Thus, to achieve optimal planning, it is important to take into account the specifics of tasks and to ensure sufficient reserves of time and resources in allocation systems.

Conclusions

The results of experimental studies have shown the importance of the optimal combination of time and resource constraints for the correct distribution of tasks between humans and robots. Analysis of successful scenarios from the first table shows that for values $T_{max} \geq 5$ and $R_{max} \geq 7$ all constraints are met, and the cost function is within 30–80. This confirms the need for balanced resources to perform complex tasks. In contrast, the second table records constraint violations with insufficient time ($T_{max} \leq 4$) and resource ($R_{max} \leq 6$) parameters, which makes task performance impossible. The graphical representation of the results emphasizes the key role of time and resources: red marks on the graph illustrate scenarios where constraints are not met, while blue dots reflect successful planning.

Comparative analysis indicates that the allocation system is robust to fluctuations in the cost function within a certain range, but sensitive to a decrease in available resources. Numerical analysis based on experimental data indicates a direct relationship between available resources and the system's ability to perform tasks. Based on the results obtained, the research prospects are to expand the model to work in variable environments, integrate machine learning algorithms to predict task parameters, and increase the system's flexibility to unpredictable loads. This will not only improve allocation processes, but also increase the efficiency and reliability of production systems in the context of Industry 5.0.

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